17MAT21

Second Semester B.E. Degree Examination, July/August 2022 Engineering Mathematics – II

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Solve:
$$(4D^4 - 4D^3 - 23D^2 + 12D + 36)$$
, $y = 0$, where $D = \frac{d}{dx}$. (06 Marks)

b. Solve
$$\frac{d^3y}{dx^3} + y = 65\cos(2x+1)$$
. (07 Marks)

c. Solve:
$$y'' + 4y = x^2 + e^{-x}$$
 by the method of undetermined co-efficients. (07 Marks)

OR

2 a. Solve
$$\frac{d^2y}{dx^2} - 4y = \cosh(2x-1) + 3^x$$
. (06 Marks)

b. Solve
$$(D^2 + D + 1)y = 1 - x + x^2$$
. (07 Marks)

c. Solve
$$\frac{d^2y}{dx^2} + y = \frac{1}{1 + \sin x}$$
 by the method of variation of parameters. (07 Marks)

Module-2

3 a. Solve
$$x^3 \frac{d^3 y}{dx^3} + 3x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} = x^3$$
. (06 Marks)

b. Solve
$$p^2 + 2py \cot x = y^2$$
. (07 Marks)

c. Modify the following equations into Clairaut's form and hence obtain its general and singular solution.
$$xp^2 - py + Kp + a = 0$$
. (07 Marks)

OR

4 a. Solve
$$(3x+2)^2y'' + 3(3x+2)y' - 36y = 8x^2 + 4x + 1$$
. (06 Marks)

b. Solve
$$p(p + y) = x(x + y)$$
. (07 Marks)

c. Solve
$$(px-y)(py+x)=2p$$
 by reducing it to Clairaut's form, by taking the substitution $X=x^2$, $Y=y^2$. (07 Marks)

Module-3

5 a. Form a PDE by eliminating arbitrary functions
$$\phi(x+y+z, x^2+y^2-z^2)=0$$
. (06 Marks)

b. Solve
$$\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$$
 for which $\frac{\partial z}{\partial y} = -2\sin y$ where $x = 0$ and $z = 0$ if y is an odd multiple of $\frac{\pi}{2}$.

c. Derive one dimensional wave equation in the form
$$\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$$
. (07 Marks)

- Form a PDE by eliminating arbitrary functions, $z = yf(x) + x\phi(y)$.
- (06 Marks)
- Solve the equation $\frac{\partial^2 z}{\partial x^2} + z = 0$ given that $z = e^y$ and $\frac{\partial z}{\partial x} = 1$ when x = 0. (07 Marks)
- c. Find various possible solution of one dimensional heat equation, by the method of separation (07 Marks) of variables.

<u> Module-4</u>

7 a. Evaluate $\int_{0}^{a} \int_{0}^{x} \int_{0}^{xy} e^{x+y+z} dz dy dx$.

(06 Marks)

b. Evaluate $\int_{-\infty}^{2} \int_{-\infty}^{x^2} (x^2 + y^2) dy dx$ by changing the order of integration.

- (07 Marks)
- Derive the relation between Beta and Gamma function as $\beta(m,n) = \frac{\Gamma m \Gamma n}{\Gamma}$. (07 Marks)

- a. Evaluate $\iint x^2 y \, dxdy$, where R is the region bounded by the lines y = x, y + x = 2 and y = 0.
 - (06 Marks)

b. Evaluate $\int_{0}^{a} \int_{0}^{\sqrt{a^2-y^2}} y\sqrt{x^2+y^2} dx dy$ by changing into polars.

(07 Marks)

(07 Marks)

c. Show that $\int x \cdot e^{-x^8} \times \int x^2 \cdot e^{-x^4} dx = \frac{\pi}{16\sqrt{2}}$.

a. Find the Laplace transform of $2^{t} + \frac{\text{Module-5}}{\cos 2t - \cos 3t}$

(06 Marks)

- b. If $f(t) = \begin{cases} t, & 0 \le t \le a \\ 2a t, & a \le t \le 2a \end{cases}$, f(t + 2a) = f(t)
 - Sketch the graph of f(t) as a periodic function and show $L[f(t)] = \frac{1}{s^2} \tanh\left(\frac{as}{2}\right)$.
- c. Find the inverse Laplace transform of $\frac{s^2}{(s^2 + a^2)^2}$, using convolution theorem. (07 Marks)

- Express $f(t) = \{1: \pi < t \le 2\pi \text{ in terms of unit step function and hence find its Laplace}\}$
 - transform. (06 Marks)
 - Find the inverse Laplace transform of $\frac{5s+3}{(s-1)(s+1)^2}$. (07 Marks)
 - Solve the differential equation $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{2x}$, y(0) = 2, y'(0) = 1 using Laplace (07 Marks) transform method.